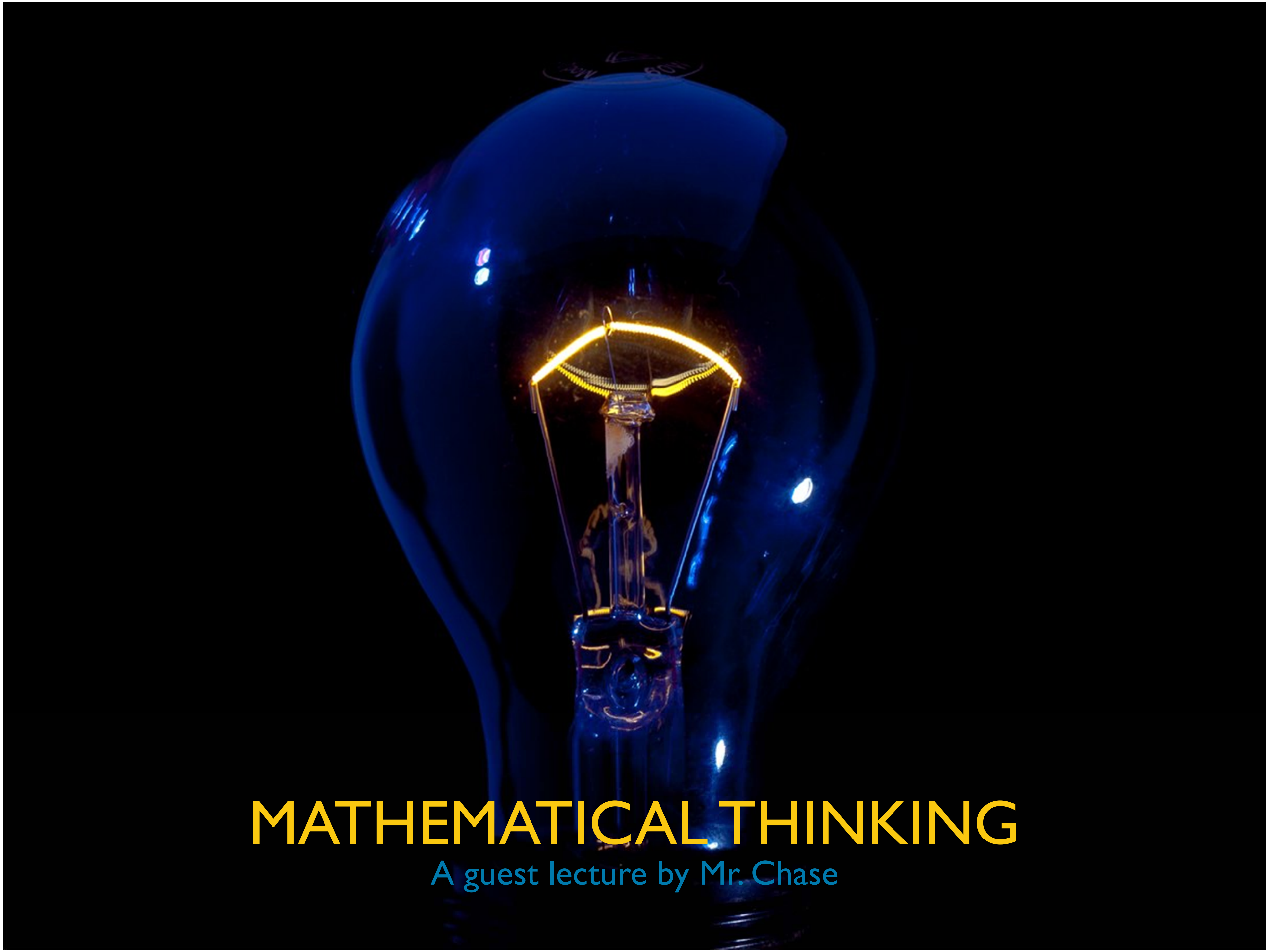
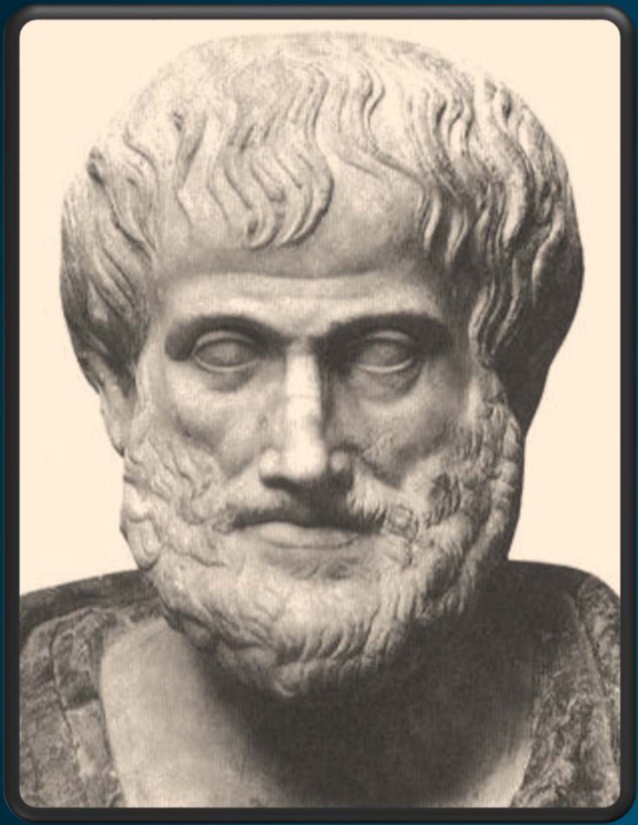
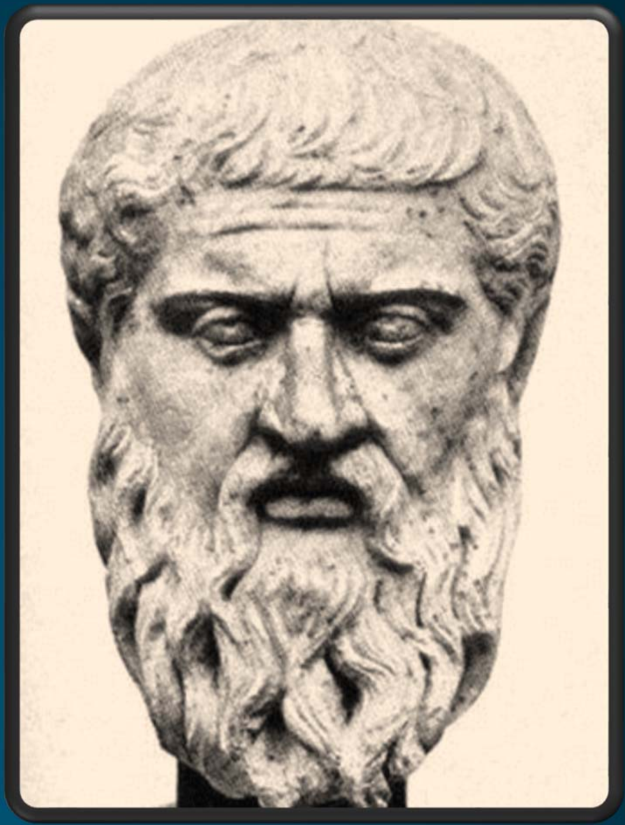

MATHEMATICAL THINKING
A guest lecture by Mr. Chase

Is mathematics invented or discovered?



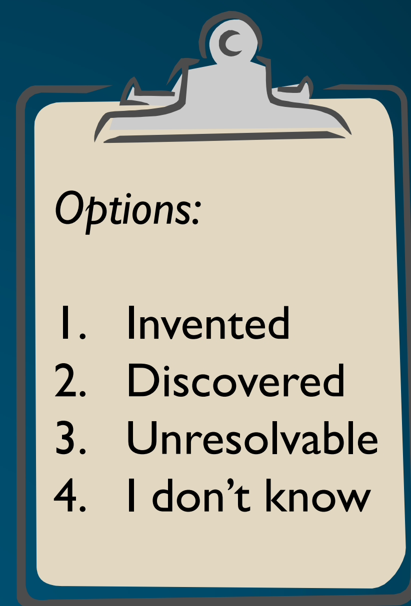
Aristotle

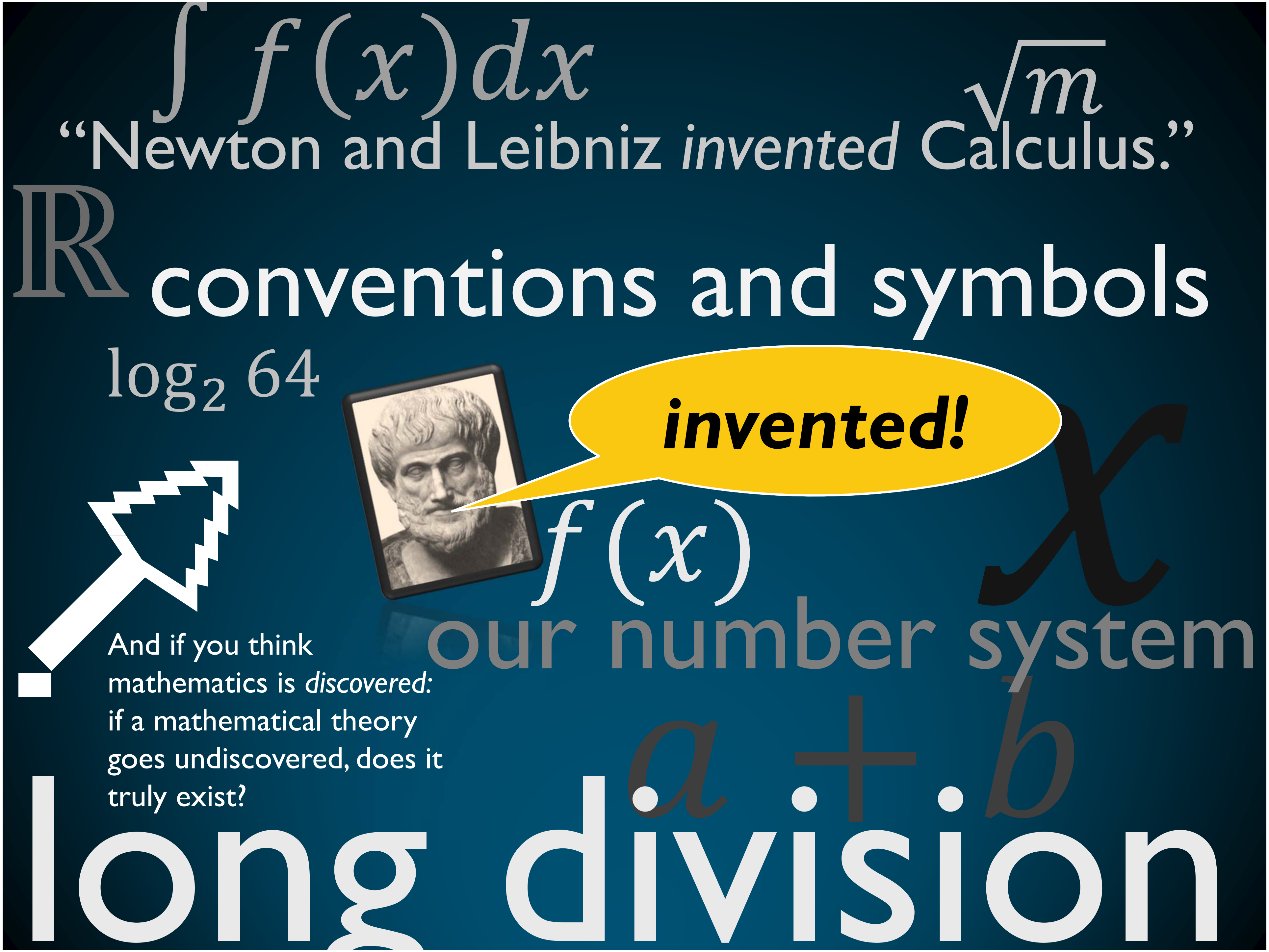


Plato

Is mathematics invented or discovered?

Poll!





$\int f(x)dx$
$\sqrt{m}$
"Newton and Leibniz *invented* Calculus."
$\mathbb{R}$
conventions and symbols
$\log_2 64$
invented!
$f(x)$
x
our number system
$a+b$
And if you think mathematics is *discovered:* if a mathematical theory goes undiscovered, does it truly exist?
long division

$\mathbb{R}$

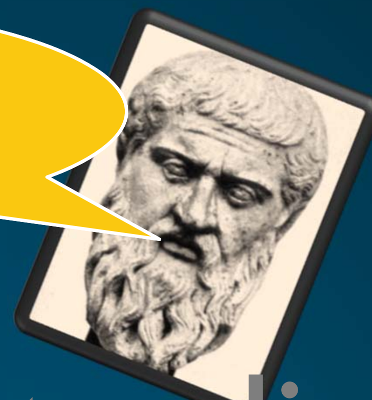
arbitrary notation

Is $2^{67} - 1$ prime or composite?

Are there an infinite number of “twin primes”?

$\int f(x)d$

$\sqrt{m}$



x

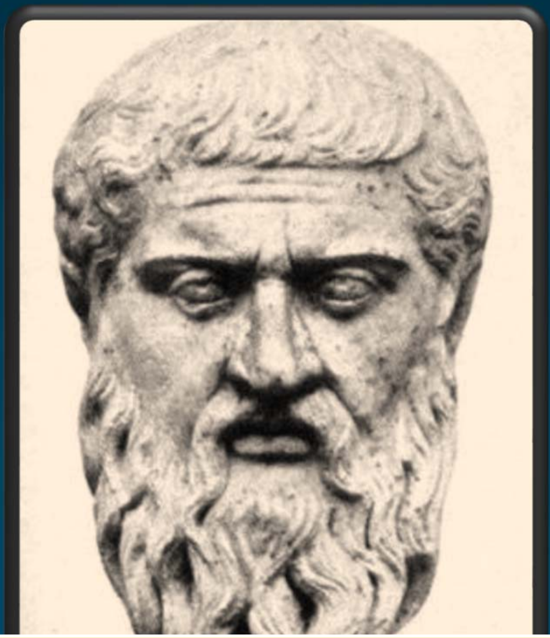
no contradictions

math is like science—
it’s true, regardless of
whether we discover
it or not.

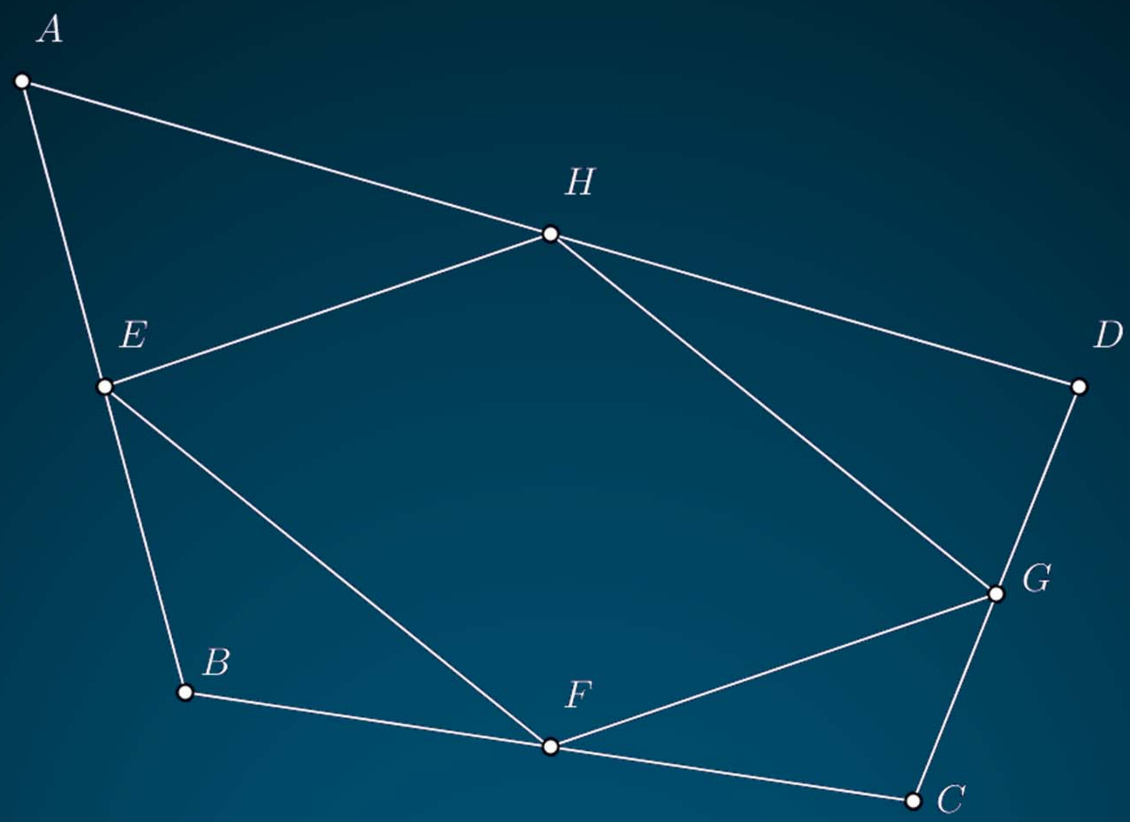
$a + b$

air-tight logic

Correct answer…



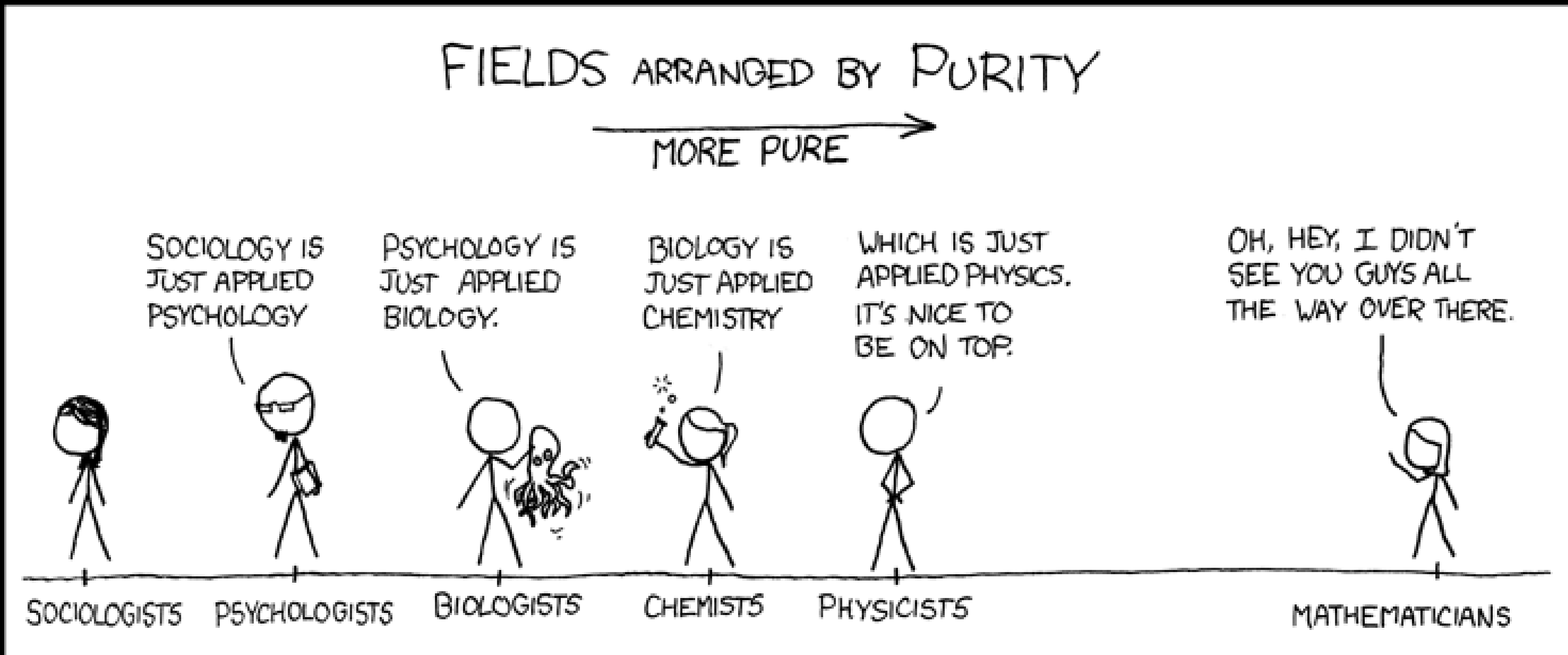
discovered!



Is this always true? Aren't you dying for a proof?

Is $9^n - 1$ always divisible by 8?

There exist two people in DC with the exact same number of hairs on their heads. Why?



“ Mathematics is a queen of science. ”

Carl Friedrich Gauss

what mathematicians have to say…

Wherever there is number, there is beauty.

Proclus

It is impossible to be a mathematician without being a poet in soul.

Sofia Kovalevskaya

The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful.

Jules Henri Poincaré

what mathematics are we free to invent?

the symbols and conventions we choose are **arbitrary**.

x

FORMALISM

Mathematics is a game played according to certain simple rules with meaningless marks on paper.

David Hilbert

the field axioms.

***Closure* of F under addition and multiplication**

For all a, b in F, both $a + b$ and $a \cdot b$ are in F (or more formally, $+$ and $-$ are binary operations on F).

***Associativity* of addition and multiplication**

For all a, b, and c in F, the following equalities hold: $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b)c$.

***Commutativity* of addition and multiplication**

For all a and b in F, the following equalities hold: $a + b = b + a$ and $a \cdot b = b \cdot a$.

See handout for some proofs based on these axioms

Existence of additive and multiplicative identity elements

There exists an element of F, called the additive identity element and denoted by 0, such that for all a in F, $a + 0 = a$. Likewise, there is an element, called the multiplicative identity element and denoted by 1, such that for all a in F, $a \cdot 1 = a$. To exclude the trivial ring, the additive identity and the multiplicative identity are required to be distinct.

Existence of additive inverses and multiplicative inverses

For every a in F, there exists an element $-a$ in F, such that $a + (-a) = 0$. Similarly, for any a in F other than 0, there exists an element a^{-1} in F, such that $a \cdot a^{-1} = 1$. (The elements $a + (-b)$ and $a \cdot b^{-1}$ are also denoted $a - b$ and a/b, respectively.) In other words, subtraction and division operations exist.

***Distributivity* of multiplication over addition**

For all a, b and c in F, the following equality holds: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$.

group

ring

domain

Can we break or change the rules?

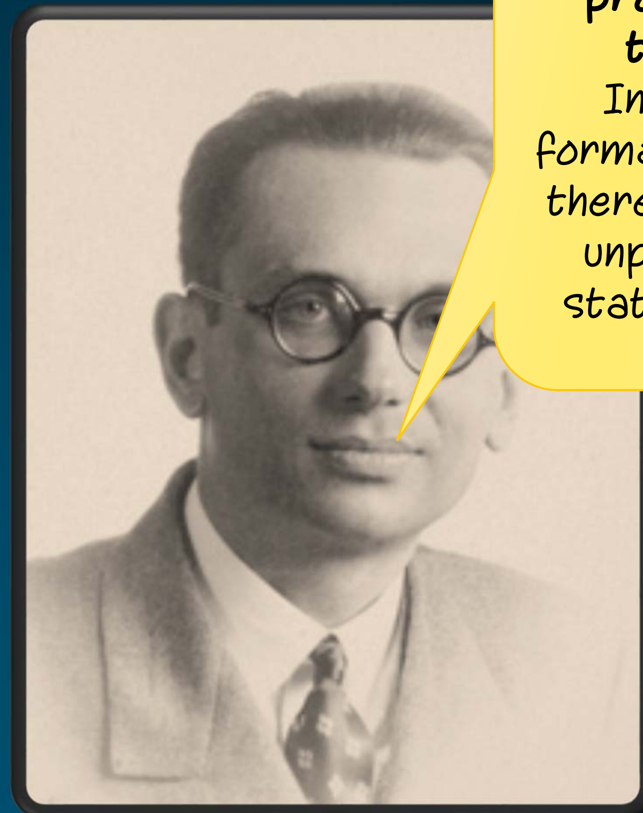
YES.

skew field

Abelian group

Epic math battles

David Hilbert



Kurt Gödel

Silly example

Axioms: it is raining outside.

if it is raining, I will take an umbrella.

Statements:

I will take an umbrella. — *Provably true.*

It is not raining outside. — *Provably false.*

I will take my pet hamster as well. — *Undecidable*

Math is useful

It's like a gorgeous painting that also functions as a dishwasher!

Ben Orlin

But…*WHY* is it useful?

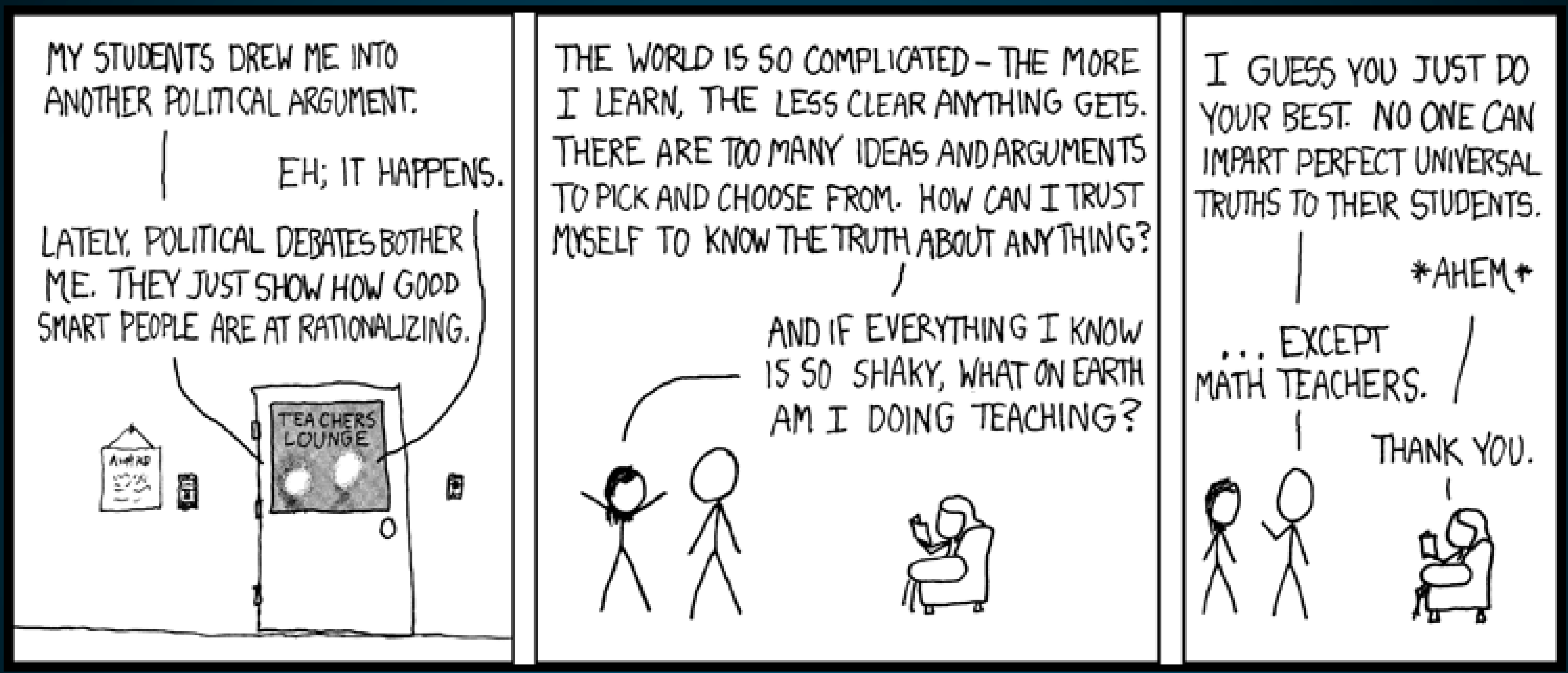
Why study math?

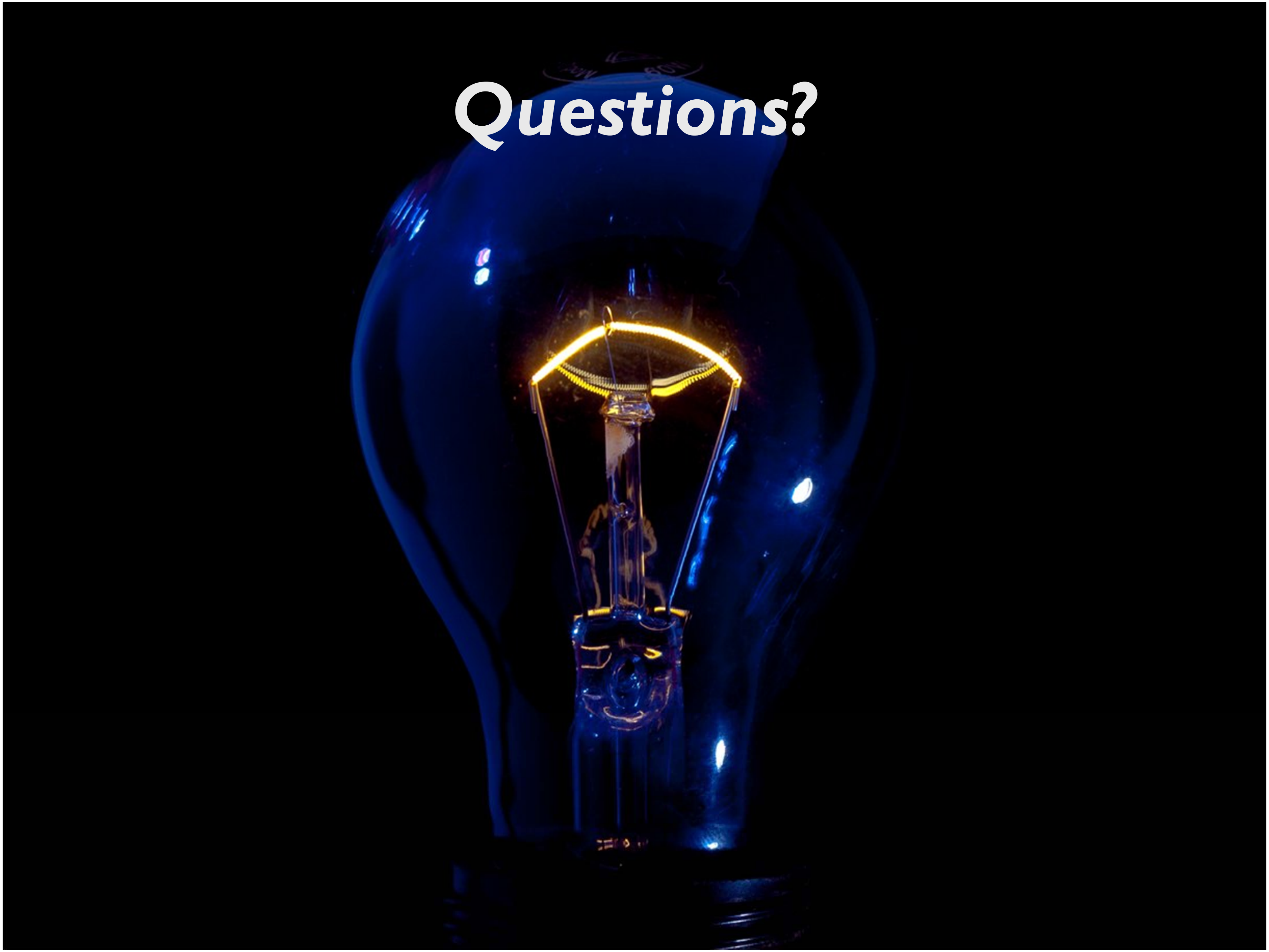
Liberal Education

Glimpsing the mind of God

In summary…

Math is different. It allows *certain* knowledge.




Questions?