TI-Nspire Giveaway Puzzle Contest

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Statement of the Problem. One can create a triangle of consecutive positive integers as follows:

1					
2	3				
4	5	6			
7	8	9	10		
11	12	13	14	15	
16	17	18	19	20	21
⋮					

Each row, R, has R numbers. Each column, C, has infinitely many numbers. Rows and columns begin at 1. We define a function $F(R, C)$ for row R and column C such that $F(R, C)$ gives us a value in the triangle. Thus, $F(1, 1) = 1$, $F(2, 1) = 2$, and $F(2, 2) = 3$.

Part 1: Come up with a formula that computes F(R,C) in terms of R and C for any positive values of R and C. Show your work.

Part 2: Come up with a formula or algorithm that, given a value n, determines R and C.

Solution. The Rth row ends with the triangular number, $\frac{R^2+R}{2}$. If n is in row R, then the *previous* row is given by

$$\begin{aligned} \frac{(R-1)^2+(R-1)}{2} &= \frac{(R-1)^2+(R-1)}{2} \\ &= \frac{R^2-R}{2} \end{aligned}$$

Adding C to get n, we have

$$n = F(R, C) = \frac{R^2-R}{2} + C \tag{1}$$

as desired.

Now, to find R and C given n, we first calculate the row. Solving $\frac{R^2+R}{2}$ for positive R gives

$$R = \left\lceil \frac{-1 + \sqrt{8n+1}}{2} \right\rceil$$

The expression inside the ceiling operator above will be an integer if and only if n is triangular. The floor of the expression would give the previous row. The ceiling will give the row n is in, so we have applied the ceiling function to ensure R is an integer. The column is then given by substituting this expression for R in (1) and solving for C:

$$C = n - \frac{\left(\left\lceil \frac{-1+\sqrt{8n+1}}{2} \right\rceil\right)^2 - \left\lceil \frac{-1+\sqrt{8n+1}}{2} \right\rceil}{2}$$